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Assessing tuning and damping of historical carillon bells and their changes through restoration

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Abstract

The 46 bells of the historical carillon in Perpignan have been restored: taking them down, sanding their oxide layer, and resuspending them. Sound and vibration recordings were made at each stage of the restoration process. The modal frequencies and decay rates have been estimated by means of the matrix pencil algorithm, a parametric signal processing method, with a variance of $\approx$ 0.1% for the frequencies and a few percents for the damping rates. Tuning is accurate in general, except for the highest notes. The quality factors of the vibration go through a broad minimum in the 2 kHz region. Measurements reveal minute but consistent frequency changes of the order of a few cents. On average, the bells ring 15% longer since the restoration. The measurements also show that the damping rates were more consistent along the range of the instrument after restoration. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

The carillon of the cathedral of Perpignan (a small city in the South of France, near the Spanish border) was built at the end of the 19th century by Bollée, a renowned founder at the time. It comprises 46 bells ranging from D4 to D8 with a few missing notes in the lower register. Most of the bells were cast for the Paris World Fair in 1870. The French Ministry of Culture decided in the early 1990′s to

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restore the carillon by setting up a fully mechanical action similar to the original one, reviewing the attachments of the bells, and sanding the 34 bells that could be removed from the carillon. The mobility of the bell tower's large shutters was also restored. On the occasion of this restoration, several questions were raised:

- how this carillon was tuned? how precisely?
- would the restoration process alter the characteristics of the bells?

In this paper, the mechanical properties of bells and the characteristics of their sound are briefly reviewed. The recording and processing techniques used to investigate the questions above are then described. Finally, the results on tuning, damping, and the comparison between measurements made before and after the restoration are presented and discussed.

2. Properties of the bells and their sound

Bells are axi-symmetrical structures which have been studied extensively. Numerous articles have been devoted to individual bells, but few to complete carillons. A complete review falls beyond the scope of this article. However, the principal papers on modes, tuning, dimensioning, and perception of bells up to 1987 have been extensively presented by Rossing in two books [1,2]. Two more recent papers review the tuning of bells [3] and the design of carillons and chimes [4] in a historical perspective.

Normal modes of a bell come in degenerate pairs which become non-degenerate when small asymmetries exist on the structure. The difference between twin eigenfrequencies is usually very small and the sound associated with a modal pair is heard as a slowly beating tone. These tones are called "partials" and their frequency is equal to the mean frequency of the two non-degenerate modal components. The partials of church and carillon bells have standard names corresponding to musical intervals. The partials are successively the Hum (1/2), the Fundamental (1), the Tierce (1.2), the Quint (1.5), and the Nominal (2), followed by an increasing density of modes [5], from which we retain the Twelve (3) and the (upper) Octave (4) because they have a strong perceptual importance. We have indicated in parentheses the ratio of the partial's standard frequency to the perceived pitch of the bell.

Since a carillon is a musical instrument, the perceived pitch of its bells relative to one another is of crucial importance. It is commonly accepted that the pitch of the strike note is that of the missing fundamental of the complex formed by three particular partials: the Nominal, the Twelve, and the Upper Octave. This evoked pitch is, therefore, roughly one octave below the Nominal. In this article, bells will be identified by this pitch, with A4 = 440 Hz as a reference.

Even though the frequency of the second partial (the Fundamental) lies very near to the frequency of this evoked pitch, it does not by itself determine the perceived pitch in most cases. When the Hum and the Fundamental sound one octave below and at the evoked pitch respectively, the clarity of the perceived pitch is of course much enhanced. The fourth partial or Quint, approximately one musical fifth above the Fundamental, often sounds very little. The third partial is normally one minor third above the Fundamental (or a sixth below the Nominal).

For psycho-acoustical reasons, this (coarse) scheme of pitch perception cannot be valid for upper bells of a large carillon. For notes with a Fundamental above C7, the Nominal is above 4 kHz where changes in our pitch perception occur. Founders probably adjust the intonation of upper and lower bells according to different perceptual schemes. It is shown in this article that this is the case with the Perpignan carillon. Hints of this difference in the tuning scheme can also be seen on the measurements made by van Heuven [6] on several Dutch carillons.

The question of decay times is of interest to many bell founders, especially when homogeneity between notes is at stake, like in a carillon. However, decay times are much more difficult to measure with precision than frequencies: beats between components can be slow enough to ruin any possibility of direct inspection or of a simple linear regression of the amplitude decay of the partial. The parametric method of signal processing that we have used overcomes this difficulty.

3. Sound recording and processing

In order to characterise the carillon and its restoration process, each bell was recorded several times. Two sets of recordings were done: one with the bells hanging in the bell tower before and after the whole restoration process, the other set with bells standing on their head at the factory, before and after sanding. This second set only comprised the 34 upper bells.

Bells were manually struck on the inner side with their own clapper, at their usual strike point which was well marked by erosion. The sound was recorded with electrodynamic microphones (AKG D24) and the vibration with a piezoelectric accelerometer (B&K 4374) coupled to a charge amplifier (B&K 2635). The accelerometer was glued on the outer rim of the bell, generally at the same angular position as the strike, or in some cases opposite to it. Before sanding, the bells were covered with a very thick layer of powdery oxide on which nothing could adhere; it was necessary to scrape the oxide layer where the accelerometer had to be glued. Sound and vibration were directly digitised, stored on a DAT recorder (Casio DA7, Sony TCD-D7, or Portadat HHB), and transferred digitally onto a PC for analysis. It was verified that the results of frequency and decay time were identical between the sound and vibration measurements. Since the latter is much less noisy (signal-to-noise ratio on the order of 50 dB), it has been used systematically in the study. For continuity of language, we will still refer to the recordings of the vibration as "sounds".

The characteristics of each sound have been extracted by the following signal processing steps:

- identify the beginning of the sound,
- measure the noise level before sound begins (to be used for the evaluation of the precision in frequency and decay time estimations),

- Fourier transform the signal and, after peak detection, roughly measure the frequency of each partial,
- isolate each partial by band-pass filtering,
- downsample each resulting signal so that about 800 points and 200 periods are kept in memory,
- estimate the frequency f and decay rate $\alpha = 1/\tau$ of each component of the partial with the matrix pencil algorithm, a parametric method for extracting exponentially decaying sinusoids in noise [7,8],
- obtain the amplitude and phase of the partial components with a least-mean-squares procedure,
- evaluate the precision of the measurement of frequencies and decay rates by computing the variances of the estimates of f and α,
- organise and store the results.

The analysis procedure has been automated up to the penultimate step. The final step requires some judgement on the relevance of the results: elimination of small partials, reordering of partials when one partial was not detected (the Quint in most cases), elimination of partials detected twice by the initial peak detection algorithm, etc. With experience, it was also possible to automate the sort procedure.

The precision in the estimations is of key importance in our case since we are interested in comparing frequencies and decay rates before and after restoration. The authors of the matrix pencil method have derived an analytical expression of the variance of the estimator based on a first order expansion of the method [7]. The variance is the same for the angular frequency $\omega = 2\pi f$ and for the decay rate α so that relative uncertainties are much smaller for the former than for the latter. For a 40 dB signal-to-noise ratio and analysis parameters given above, the frequency uncertainty is below 0.1% which represents 1/60 of a semitone. The decay rate uncertainty is a few percent. This uncertainty evaluation was confirmed by the dispersion of the values of f and α estimated on several partials.

When two components are present, the variance of the matrix pencil estimator is close to the Cramer-Rao boundary [9]. The CR boundary sets a theoretical limit to the estimation precision for a given signal-to-noise ratio and a given number of data points retained for analysis [10]. The value of the CR limit decreases with the number of points taken in the analysis. For a given computing power and storage capacity, one may prefer a simpler method to the matrix pencil algorithm. When one component only is present, the Hilbert analysis, for example, can be used. Since it is less computationally demanding, it can be performed on more points and would therefore approach a lower CR limit. However, the Hilbert analysis does not approach the CR limit as closely as the matrix pencil method. The various factors influencing the compromise will be presented in a forthcoming article.

Given the frequent occurrence of non-degenerate pairs of modes, a parametric analysis is necessary. The matrix pencil method represents one of the best available choices. For the purpose of analysing a large number of bells and partials, we developed an automatic treatment.

4. Tuning

The deviation from equal temperament of the six principal partials is displayed in Fig. 1. In this figure and all other ones, results are indicated by a "o" mark for isolated notes, either because adjacent notes are missing in the carillon or because the parameter could not be estimated on the adjacent notes for that particular set. For each of the six partials, four curves are plotted corresponding to the two frequencies present in each partial, both before and after restoration. Deviation from equal temperament is shown in cents (a cent is a hundredth of a semitone). The frequency f_{par} of each partial is normalised as 1200 $\log(f_{par}/f_{norm})/\log(2)$. The reference frequency is $f_{ref} = 868$ Hz and corresponds to the average of the Fundamental on all notes excluding the 10 top ones. The normalising frequency f_{norm} for each partial is:

$$f_{norm} = f_{ref} r_{note} r_{par} \quad (1)$$

The interval ratio r_{note} is computed for the nominal pitch of the bell according to equal temperament. The partial ratio r_{par} is 0.5, 1, 1.2, 2, 3 and 4 for the Hum, Fundamental, Tierce, Nominal, Twelve, and Upper Octave respectively. For example, f_{norm} for the Nominal of B5 is $868 \times 2^{2/12} \times 2$.

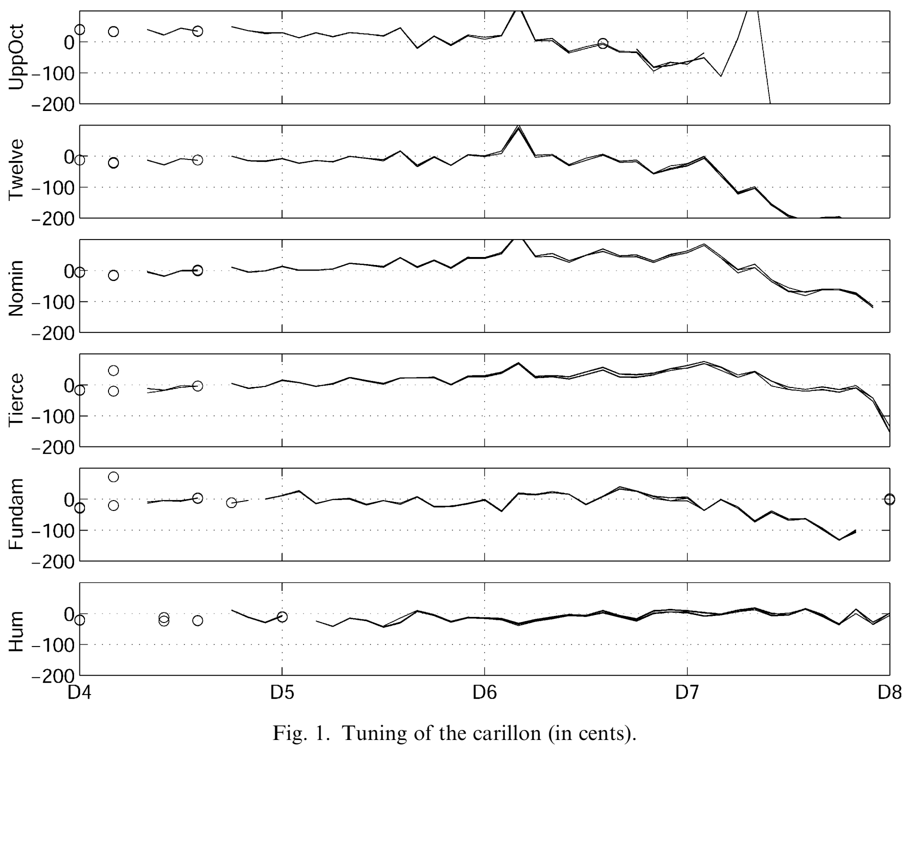

Fig. 1. Tuning of the carillon (in cents).

Some notes are missing in the carillon in the low register and the Hum was not properly detected for a few low bells. This is not really surprising with the one-strike excitation that was given to them. Contrary to upper and lighter bells which are fixed, the lower bells are freely swinging. One realises by listening to these low notes that it takes several strikes to give the lowest partial a significant amount of energy.

The curves in Fig. 1 are nearly indistinguishable which means (a) that the eigenfrequencies in each doublet differ very little and (b) that restoration did not affect the frequencies significantly. The latter is analysed with more details in the next section.

As far as tuning is concerned, a D7–E7 limit defines two registers. Below this limit, the Tierce and the Nominal follow the same trend and also the same variations from note to note. Averaged on the notes between D4 and E7, the ratio between the frequency of the Tierce and half the frequency of the Nominal is 1.195. On average, this puts the Tierce 9 cents above a tempered minor third (1.189) and 7 cents below a natural minor third (1.2). However, the concept of "natural" third is meaningful only for a chord of two harmonic tones: their common harmonics would not beat in the case of a natural third. The normalised frequencies of the Tierce and the Nominal increase with pitch: between B4 and D♯7, the trend is 24 cents per octave for the Tierce and 29 cents per octave for the Nominal. In a slightly shifted range, the trend is 17 cents for the Hum. This is of the order of magnitude of perceptual octave stretching at this pitch range which suggests that the tuning was rather carefully done on these two partials. We have found no clues in the tuning pattern of a preference for another temperament than the equal temperament in use at that time. The other partials do not follow this trend for octave stretching.

The tuning pattern changes dramatically above E7. For these notes, the frequencies of the Nominal exceed 5 kHz which represents the limit for a precise perception of tonal pitch. Therefore, it is to be expected that the complex of the Nominal, the Twelve, and the Upper Octave no longer determine the pitch. Only the Hum remains in tune with a tempered scale. The Tierce is tuned according to a natural third, except for the highest note and at the junction between the two registers. All other partials fall flat. Listening to the group of ten or so upper bells reveals that they do not sound as nicely in tune as the other ones.

5. Changes in frequencies brought by the restoration

The frequency variation due to sanding is extremely small. Two comparisons were made: one between sounds recorded at the factory before and after sanding and one between sounds recorded in the bell tower before and after the restoration work. In both cases, the frequency changes were fairly uniform.

On average, the frequency decreased by 4 cents in the first comparison set. This must be corrected by the effect of a 10 °C temperature increase between the two recording sessions. According to van Heuven [6], a 1 °C increase in temperature causes a 4×10^{-4} relative decrease in the frequencies of bell partials. Therefore, the sanding process itself increased the frequency by about 3 cents i.e. 0.18%. This is below the 4 cents (or more) auditory threshold for detecting a pitch variation.

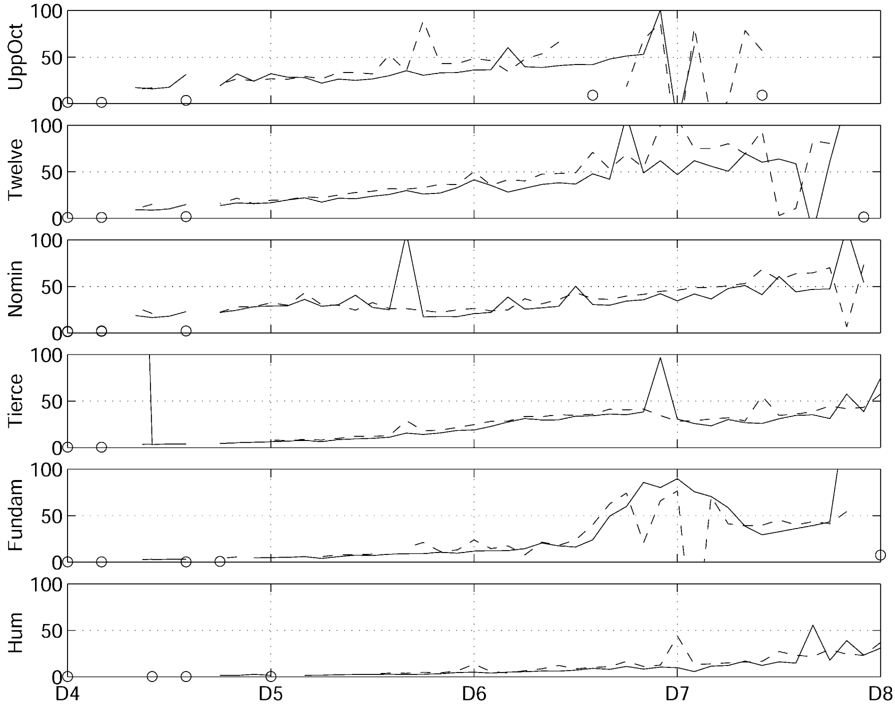

Fig. 2. Decay rate of the main component of each partial, in dB/s. Solid: after restoration, dash: before restoration.

This frequency increase is consistent with the removal of the superficial oxide layer by sanding. Since this small quantity of matter was not in solid state, it did not contribute to the elasticity of the bell but only to its mass. Changing the mass without changing the elasticity is effectively the same as changing the density of a homogeneous shell. Since the partial frequencies go like the inverse of the square root of the density, one can infer that the quantity of matter that has disappeared by sanding is 0.36%.

The oxide layer that has been removed by sanding was solid bronze at the time of casting and so originally did contribute to the elasticity of the bell. The transformation of the superficial bronze into oxide followed by sanding has had the net effect of decreasing the thickness h by 0.36%. The frequencies of partials are proportional to h^{α} with α being 0.7 for the Hum and 0.85 for the other partials considered here [11]. One can conclude that originally, at the time of casting, the partial frequencies were 5 cents higher than what they are now, after restoration. Before restoration, they were 8 cents below what they were at the time of casting. Both numbers are hardly significant in terms of pitch perception and less than what occurs after a typical seasonal temperature change.

In the second comparison set (measurements in the bell tower, before and after restoration), the frequency change amounts to −1 cent on average. The effect of the temperature change between the two recording sessions was −2 cents. Therefore, the restoration is by itself responsible for a +1 cent net increase. Since sanding had a +3 cents effect, −2 cents must be attributed to the difference in the tightening of the bells to their support after they have been resuspended. Again, this is an average value.

6. Damping and its changes over restoration

The decay rates for the main components in each bell partial are presented in Fig. 2. The decay rates for the other component are very similar but they are not shown for the sake of clarity. As expected, upper partials decay faster than lower ones. For comparison, a reverberation time of 2 s, typical for a concert hall, corresponds to $\alpha = 30$ dB/s. In a church, the reverberation could be much longer, of the order of 10 dB/s or less.

A close examination of the decay rates before and after restoration reveals that there are less fluctuations in the curves after restoration. This is probably related to the systematic retightening of the bells on their support.

When one component is much smaller than the other one, the precision of estimation becomes poor for that component. In such a case, beats would not be heard.

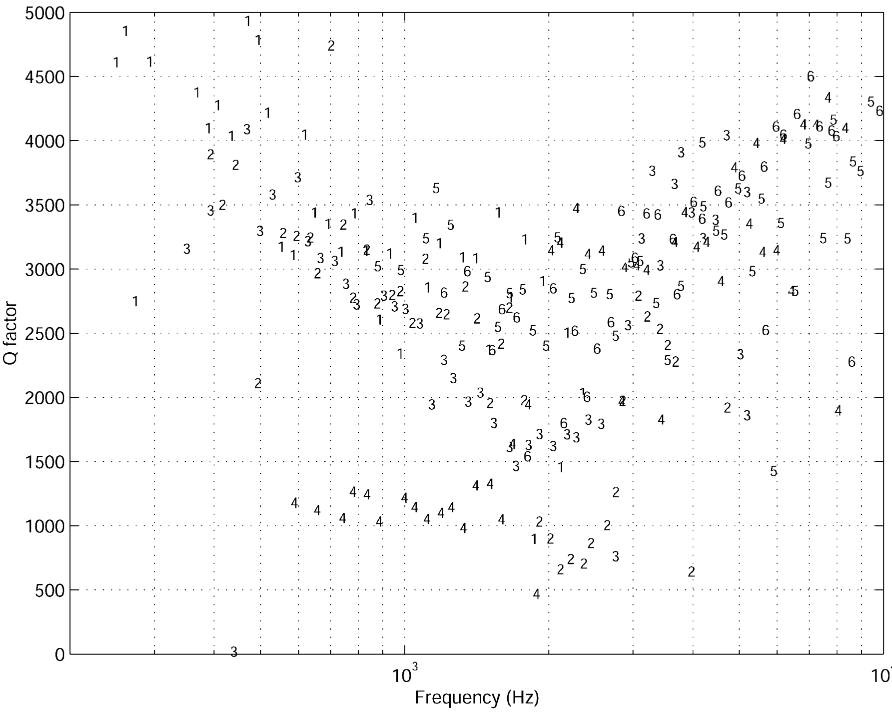

Fig. 3. Q-factor of the main component of each partial (1 for Hum, ... 6 for Upper Octave) as a function of frequency (not pitch). Data are taken after restoration.

The sudden large damping of the Fundamental around the note C7 (Fig. 2) has no clear origin. This was also observed when the bells are measured in the factory, with their head on the floor. This phenomenon is therefore intrinsic to the bell and cannot be attributed to coupling with a resonating structure in the bell tower.

The *Q*-factors ($Q = \pi\tau f$) of the main component of each partial of each note are presented in Fig. 3 as a function of frequency. Despite the large scattering of the data relative to each partial, in general there is a clearly visible dip in the value of the *Q*-factor in the region of 2–3 kHz. This is the frequency region favoured by the human ear. Investigating further on perceptual matters would need an amplitude reference from bell to bell which we did not get. In any case, the similarity of the *Q*-factors of different partials is a strong indication that the mechanisms of internal damping and sound radiation are similar for all partials of the different bells.

As shown in Fig. 2, the vibration typically has a longer decay rate after restoration of the bells. Sanding is mainly responsible for this variation of 15% on average for all notes. This is certainly audible although not a large effect.

7. Conclusion

Modal frequencies and damping rates of the 46 bells of the historical carillon in Perpignan have been studied over a restoration process. The precision required for assessing the small changes induced by the restoration was assured by the matrix pencil algorithm. This parametric signal processing method also overcomes the practical impossibility of measuring decay rates of beating tones by conventional methods such as Fourier transform, direct inspection of time envelope, and Hilbert analysis. The entire signal processing procedure has been made automatic, including the first step of locating and isolating each partial and the final one of sorting the results for all notes and partials. This procedure can now be transformed into a useful tool for professionals of bells and carillons.

On the perceived pitch of the notes, we found that the carillon was most probably tuned according to equal temperament. The tuning is done on the Nominal up to note E7 and was probably done on the Hum above E7. The relative tuning of partials is good in the low and middle registers but becomes much less precise for the treble notes.

Since the cast about one century ago, an oxide layer had developed on the bells which caused the partial frequencies to decrease by 8 cents on average (slightly less for the Hum). Of which, 3 cents have being regained by sanding and eliminating the oxide layer. One can, therefore, estimate that approximately 0.4% of the bronze was transformed into oxide and removed. The order of magnitude of the frequency changes caused by temperature variations is also a few cents. All these changes are perceptually insignificant.

The decay rates vary from a few dB/s to a few tens of dB/s, increasing regularly with the pitch of the notes and with the rank of the partial. The representation of damping in terms of *Q*-factors reveals that all the partials follow a similar pattern: decreasing from a value of roughly 4500 at low frequencies to 2500 at 2 kHz and increasing again to 4000 at 10 kHz. This curve has some similarity with the ear sensitivity: bell tones sound shorter at frequencies to which the ear is most sensitive. This might suggest that the metallurgy and design of these bells would somehow compensate for the ear characteristics, unlike the case of violins or opera singers, for example, where this frequency range is emphasised.

We observed an average 15% decrease of the decay rates of the partials of all notes. This increasing duration of the notes is audible but not an important effect. Two factors in the restoration process could have influenced the decay rates: sanding and retightening of the bells on their support. The large variance of changes in the decay rates before restoration is consistent with the observation that the restoration process had the effect of partly smoothing out the decay rates over the notes. This suggests that a regular tightening of the bells on their support is important in order to obtain a regular sound over the carillon playing range.

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