SHAPE OPTIMIZATION OF CARILLON BELLS

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Abstract

The sound spectrum of a bell is the total audible sound that is radiated by the vibrating bell and is a superposition of a large number of tones, the so-called partials. Each partial is radiated by a single structural eigenmode and can be characterized by its frequency, strength and decay speed. The dynamic response and the acoustic damping of the bell are calculated using finite elements and boundary elements, respectively. Then the sound spectrum characteristics of a major third bell are optimized by means of the sequential linear programming method.

1. Introduction

The sound spectrum of a bell consists the superposition of a large number of partials or overtones, each with a distinct frequency. The partial frequencies are determined uniquely by the shape and material of the bell. During the slow decay of the sound spectrum the partial frequencies remain the same since the weakly damped bell behaves in a linearly elastic manner. In Fig. 1a a traditional North European carillon bell system is shown.

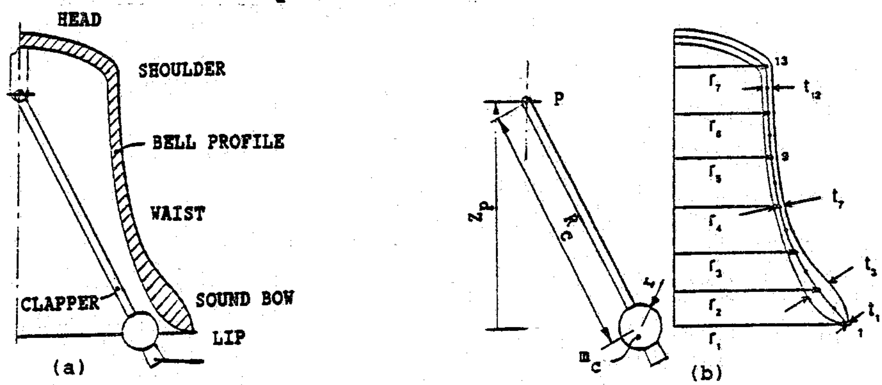


Fig. 1 a. Carillon bell system; b. Bell system design variables

Every eigenmode vibrates with its own eigenfrequency in a unique modeshape, has an initial amplitude that is determined by the initial conditions, excitation process and excitation point, and decays with its own decay rate due the damping related to the eigenmode. The eigenfrequency, modeshape, damping and amplitude of an eigenmode are called the modal properties. For an axi-symmetric bell as shown in

Fig. 1a each eigenmode can be characterized by the combination of one vibration mode in the horizontal and one in the vertical cross-section, shown in Fig. 2. The dotted lines display the undeformed shape of the midplane of the bell.

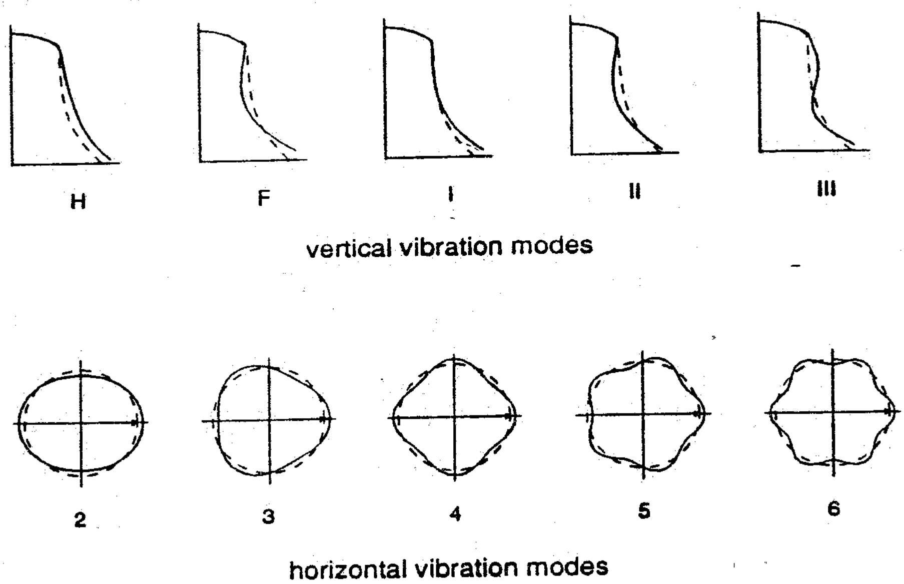


Fig. 2 Vibration modes of a traditional North European bell.

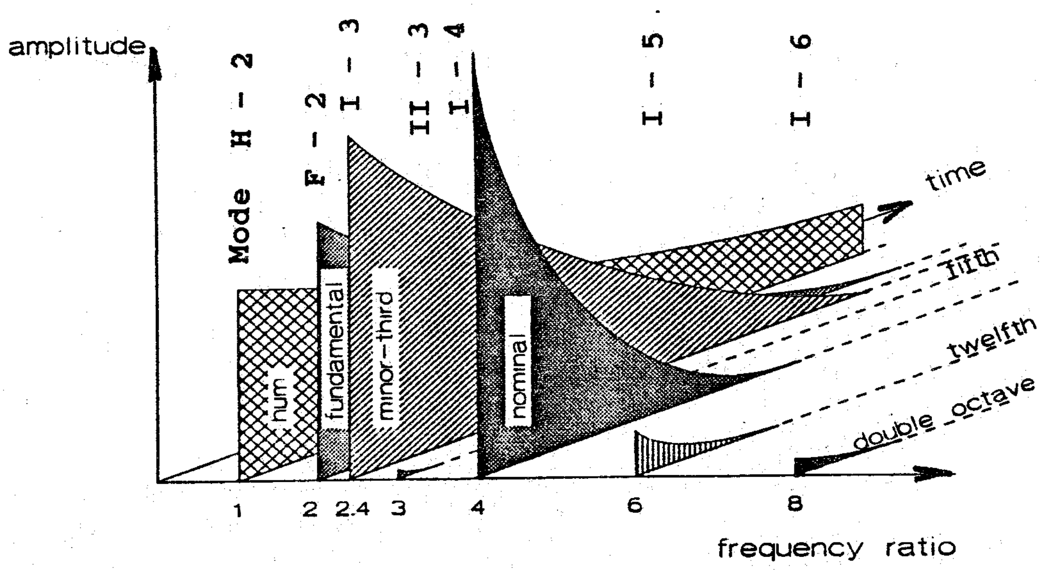


Fig.3 Schematized sound spectrum of a minor third bell.

Every eigenmode excites the air surrounding the bell, generating a vibration in the air with the same frequency. This vibration propagates through the air (away from the bell) generating the sound field. Not all partials in the sound spectrum are equally important. To be important partials have to be loud and of low order. The frequency ratios of these partials constitute the overtone structure. For a traditional North European minor third bell the names of the lowest and loudest partials, their frequency ratios, and eigenmode codes (according to Fig. 2) are given in Fig. 3.

The subject of this paper is the modelling and optimization of the sound spectrum of bells. More specific, the frequency, strength and decay rate of the limited number of important partials in the sound spectrum, are modelled and optimized with respect to the geometry of the bell and the clapper. The geometry parameters describing the shape of the bell and the clapper are used as the independent variables in the modelling stage, and as the design variables in the optimization stage, see Fig. 1b.

2. Contact model and dynamic response calculation

The dynamic response of the bell is calculated by a Finite Element (FE) code, [1], using axi-symmetric elements with non-axi-symmetric deformations. Fig. 4a shows the applied finite element mesh. The dynamic response of the bell can be determined if the excitation of the bell is known. This requires knowledge of the contact force between bell and clapper. However, the contact force during the collision is unknown beforehand. Therefore the model of the contact phenomenon has to be combined with the FE model of the bell.

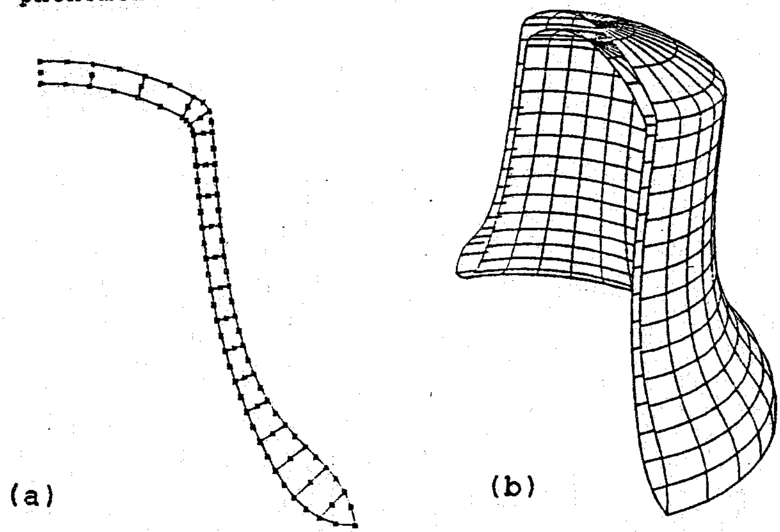


Fig. 4 a. FE-mesh for the structural response calculation
b. BE-mesh for the sound pressure calculation

The compression $\alpha(t)$ between the bell and the clapper in the contact point, perpendicular to the bell wall, is assumed to be caused by the normal component $F_n(t)$ of the contact force only, see Fig. 5. Further it is assumed that the relation between $F_n(t)$ and the compression $\alpha(t)$ can be described by Hertz's law

$$F_n(t) = k(\alpha(t))^{1.5} \tag{1}$$

where k is a constant depending on the geometry and material properties of the colliding bodies. The compression $\alpha(t)$ perpendicular to the bell wall in the contact point C is defined as the difference between the displacement $w_{cl}(t)$ of the clapper and the displacement $w_b(t)$ of the bell in contact point C, both normal to the bell wall, see Fig. 5.

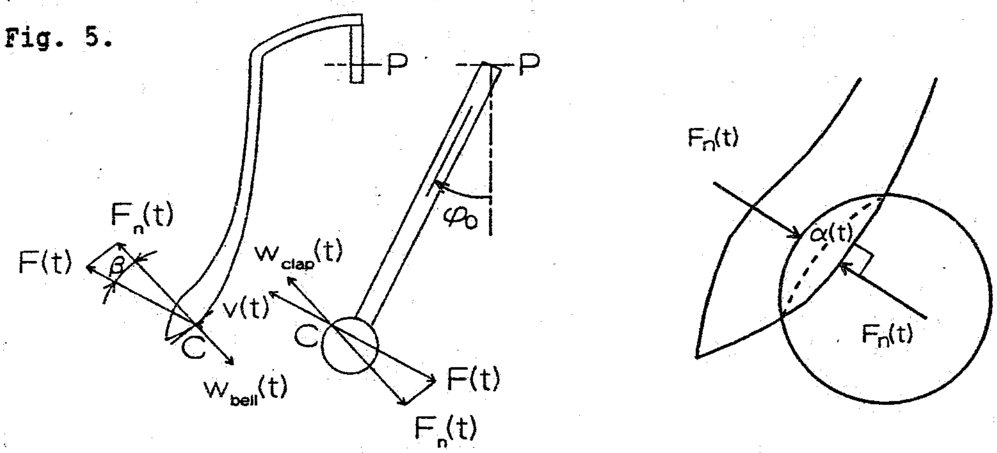


Fig. 5. Contact model of bell and clapper

Using the finite element model of the bell the displacement $w_b(t)$ can be expressed in the unknown contact force F(t). In a similar way the displacement of the clapper $w_{cl}(t)$ can be expressed in F(t), using the equation of motion of the clapper. The obtained expressions for $w_{cl}(F(t))$ and $w_b(F(t))$ are substituted in Eq. (1)

$$F(t)\cos(\beta) = k\{w_{cl}(F(t)) - w_b(F(t))\}^{1.5} \tag{2}$$

Eq. (2) is a non-linear equation in the unkwown contact force F(t) and can not be solved analytically. Therefore Eq. (2) is discretized in time and F(t) is solved numerically. At the same time the displacements of the clapper and the bell are calcultated from $w_{cl}(F(t))$ and $w_b(F(t))$ respectively. The described modelling and calculation of the dynamic response could be validated by measurements on a real bell system, [1].

3. The modal damping; Fast analysis models

In common bell systems the damping of the eigenmodes is very small. Therefore the damping can be neglected in the dynamic response calculation as described in the previous section. The initial strength of the overtones, see Fig. 3, will hardly be influenced by the small damping.On the other hand the modal damping is essential for detereming the decay rate of the partials.Due the damping the vibrational energy of the eigenmodes decreases exponentially in time according to

$$E_i(t) = E_i^o \exp(-2\eta_i\omega_i t) \qquad (3)$$

where η_i is the modal damping and ω_i is the angular eigenfrequency.

The modal damping is subdivided into material damping and acoustical damping which are of the same order of magnitude. The material damping is defined as the dissipation of vibrational energy into heat due to internal friction mechanisms in the bell material. The acoustical damping is defined as the decrease of vibrational energy of the bell due to the radiation of sound energy into the air surrounding the bell. This damping component depends very strongly on the modeshape and the eigenfrequency of the radiating eigenmode, and on the geometry of the bell.

Material damping parameters can be found in literature or can be measured on bells placed in a vacuum vessel. We calculated the acoustical damping using the boundary element (BE)software code SYSNOISE, [2]. Therefore a three-dimensional BE-mesh is needed, see Fig. 4b. Due to the large mesh these computations are very time consuming: one eigenmode requires about 3 hrs CPU on an Alliant FX4 computer. Therefore we computed "off line" the acoustical damping parameters of about 300 different shaped bells, fitted approximating damping models on the results, and we used these approximating models in the optimization process.

Similarly we built approximation models for the eigenfrequencies as function of the bell geometry by fitting eigenfrequency values which are computed with the FE code mentioned in Section 2, [1]. Thus we created "fast" analysis models for both the eigenfrequencies and the acoustical damping of the most important eigenmodes of the bell.

4. Optimization

In this section the developed software is used to optimize major third bells. In 1985 we designed the first major third bell, [3]. Such a bell has the frequency ratios 1 : 2 : 2,5 : 3 : 4 for the lowest five eigenfrequencies. If the bell is struck with the clapper a strong major third chord is generated due to the ratio, 2.5, of the third eigenfrequency. In traditional West European carillon bells this ratio is 2.4, resulting in minor third bells.

However, the design of the major bell not only meant the change of one eigenfrequency, but other modifications of the bell timbre occurred as well. We have (at least) two options to proceed. The first option is to regard the major carillon as a complete new musical instrument, and to optimize the sound of this instrument. Therefore a lot of perceptual investigations has to be done in order to find the optimization goals. In the second option it is tried to design major third bells with a timbre that, except of course the ratio 2.5, resembles the timbre of the minor third bell as much as possible.The minor bell timbre is well known, and optimization goals can be formultated more easily than in the first option. That is the reason why we proceed with the second option. However, it should be emphasized that both options are legal.

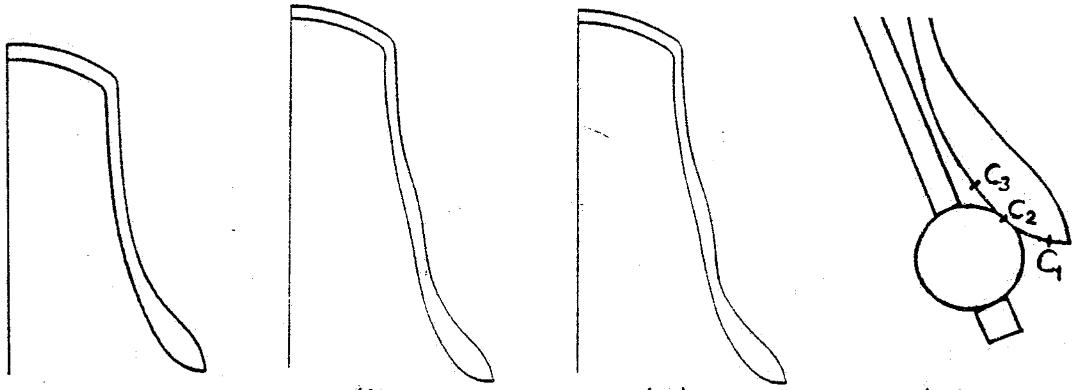
In the optimization process we manipulate the bell system design variables, see Fig. 1b, in such a way that a certain objectieve function is minimized subject to a number of constraint functions. The applied objective function is defined as the weighted sum of squared deviations of response parameters (eigenfrequencies and damping parameters) from the according target values. In the first stage of the optimization process only the bell geometry is varied, and we used the fast analysis models to calculate frequency and damping values. In the second stage the obtained solution is taken as a starting geometry for an optimization process, where the frequencies are directly computed by the FE-program, while the damping parameters are still taken from the fast analysis models. Finally, in the third stage the optimized bell geometry is kept constant and used to find and optimal strength of the overtones by variating the position of the contact point between the bell and clapper.

As constraint functions only upper and lower bounds on the design variables are used preventing the geometry to become infeasible.

The optimization problem is solved by means of the well-known Sequential Linear Programming (SLP) method. In each iteration step the optimization problem is linearized and a sequence of linear programming subproblems is solved until convergence is reached.

5. Results and conclusions

The fast analysis models enable us te scan the design area with a little computational effort. Therefore a number of starting geometrics are randomly chosen within the design area, and then the bell is optimized. In this paper we present the optimization process starting from a bell geometry which is close to the geometry of the traditional minor third bell, see Fig. 6a. The result of the fast optimization is given in Fig. 6b. This geometry is taken as the initial geometry for the FE-based optimization of the frequency ratios; the damping values are still optimized using the fast analysis models for the acoustical damping. Fig. 6c shows the final bell shape. The numerical results are given in Table 1.



(a) (b) (c) (d)

Fig.6. a. Initial geometry; b. Result of fast optimization;
c. Result of FE-based optimization;
d. Optimization of the contactpoint.

In this case-study we assumed the material damping to be constant on the value $0.9*10^{-4}$, which represents a bell bronze of usual quality.

Partial name	Target values		Initial geometry		Results using: Fast analysis		Results using: FE-analysis	
	freq. ratio	η_{tot} $*10^{-4}$	freq. ratio	η_{tot} $*10^{-4}$	freq. ratio	η_{tot} $*10^{-4}$	freq. ratio	η_{tot} $*10^{-4}$
Hum	1.000	1.11	1.000	1.21	1.000	1.13	1.000	1.13
Fundamental	2.000	1.64	2.023	1.54	2.055	1.57	2.004	1.46
Third	2.520	1.62	2.380	1.78	2.528	2.03	2.510	1.83
Fifth	2.997	3.71	3.143	2.80	3.048	2.63	3.003	2.68
Nominal	4.000	3.82	4.008	4.02	4.157	4.03	4.017	3.63
Twelfth	5.993	1.61	6.027	1.65	6.100	?	5.861	1.65
Double oct.	8.000	1.55	8.336	1.67	8.351	1.82	8.029	1.90

Table 1. Frequency ratios and total damping values for the shape optimization of the bell.

The initial damping values are already close to the target values, because we chose the minor third bell as the initial geometry. The initial frequency ratios are quite far from the targets, but they are very much improved in the fast optimization, while globally preserving the quality of the damping values. The FE-based optimization again improves the frequency ratios at somewhat improved damping values.

In minor third bells the contact point position is determined experimentally, choosing the contact point that yields the "best" bell sound. In practice the fifth is always excited very weakly. Numerically optimizing the contact point position of the major third bell three options can be distinguished. The first option is to disregard the strength of the fifth, placing all emphasis on the remaining six partials. In the second option it is attempted to reach the target value of the fifth, giving little importance to the remaining six partials. The third option is not to excite the fifth at all, disregarding the remaining six partials. In Table 2 the target values and the sound power ratios, corresponding to the contact point positions that result from the three options are listed.

Partial name	Target Power ratios	Power ratios C_1	Power ratios C_2	Power ratios C_3
Hum	1.000	1.000	1.000	1.000
Fundamental	16.946	1.383	0.800	0.428
Third	30.371	7.664	5.503	4.156
Fifth	0.356	2.353	0.503	0.000
Nominal	26.158	43.307	26.631	18.411
Twelfth	14.313	58.407	32.335	23.425
Double oct.	29.242	26.613	13.231	11.392

Table 2. Sound power ratios from the contact points C_1, C_2 and C_3

Employing the first option, it is found that the optimization algorithm places the contact point as low on the sound bow as possible. This is caused by the fact that (except for the fifth) none of the eigenmodes has a nodal circle near the lower part of the sound bow. The resulting contact point C_1, see Fig. 6d, is not very practical, however. Trying to reach the target value of the fifth, while placing little emphasis on the remaining six partials, the contact point C_2 is obtained, which is situated on the sound bow, and can used in practice. Employing the third option, i.e. (practically) no excitation of the fifth, the contact point C_3 is located on the nodal circle of the fifth. From Table 2. it can be seen that for all three options the target values have not been reached. Especially the strength ratios of the lowest partials (i.e. the fundamental and the third) are not even near the target values. It is concluded that there is no contact point on this major third bell that yields approximately the same sound power ratios as the minor third bell.

References

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[2] SYSNOISE 4.3 Users manual, NIT, Leuven, Belgium, 1990.

[3] Schoofs, A., Asperen, F. van, Maas, P., Lehr, A., "Computation of bell profiles using structural optimization", Music Perception, vol.4, no. 3, pp. 245-254, 1987.